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A Complete Set of Field Equations For the Dynamic Simulation Of a Towed Cable System

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PREFACE

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The development of a complete set of field equations describing the dynamic behavior of a towed cable is presented. The model includes inertial effects, as well as bending, torsional, and extensional rigidity. The resulting field equations are a "generic" set of 13 coupled, nonlinear, partial differential equations in time and unstretched cable scope. Hydrodynamic loading and "added mass" are left as user-defined functions.

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LIST OF ILLUSTRATIONS

LIST OF SYMBOLS

1. English Letters

A_1 A_2 A_3 A_4	Cable bending and torsional constants, defined in (38) and (39).
$\hat{b}$	Binormal unit vector of flow-based coordinate system, defined in figure 5.
$\hat{B}$	Binormal unit vector of curvature-based coordinate system, defined in figure 6.
$\vec{D}$	Drag and hydrodynamic loading force per unit length, defined in figure 7 $\vec{D} = D_n \hat{n} + D_b \hat{b} + D_t \hat{t}$.
e	Cable stretch, defined in (16) and (17).
E	Effective Young's Modulus of cable.
G	Effective Shear Modulus of cable.
I_0' I_1' I_2' I_3'	Principal moments of inertia of cable per unit length, as defined in equations (31) and (32).
I_B	Cross-sectional bending moment of inertia, (40).
I_P	Cross-sectional twisting polar moment of inertia, (37).
ℓ_0, ℓ	Arc length along unstretched and stretched cable.
M_1 M_2 M_t	Bending and twisting moments in cable around the $\hat{1}$, $\hat{2}$, and $\hat{3}$ axes, as shown in figure 8.
M_b	Total bending moment in cable owing to cable curvature, as shown in figure 9.
$\hat{n}$	Flow-based normal unit vector, figure 5.
$\hat{N}$	Curvature-based normal unit vector, figure 6.
R	Local radius of curvature of cable.
$\vec{r}$	Vector from inertial origin to cable point.
$\hat{t}$	Flow-based tangent unit vector (same as $\hat{3}$).
$\hat{T}$	Curvature-based tangent unit vector (same as $\hat{3}$).
T	Cable tension.

LIST OF SYMBOLS (Cont'd)

1. English Letters (Cont'd)

$\vec{u} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$	Cable velocity in inertial reference frame in the x, y, and z directions.
$\vec{u}_c$	Local current.
$\vec{U} = \begin{Bmatrix} U \\ V \\ W \end{Bmatrix}$	Cable velocity with respect to water $\vec{U} = \vec{u} - U_c$.
$V_1\ V_2$	Shear forces in 1 and 2 directions.
w_0, w	Weight per unit length of the unstretched and stretched cable.

2. Greek Letters

α	Angle between cable tangent vector and flow, figure 5.
ε	Cable strain e (stretch) = $\varepsilon + 1$.
θ	Euler's second angle, see figure 3.
$\mu_0\ \ \mu\ \ \mu_a$	Mass per unit length of unstretched and stretched cable, and added mass.
ρ	Local cable curvature = $1/R$, also water density.
σ	Normal stress.
$\vec{\tau}$, τ	Torques in principal axis coordinate system, shear stresses.
φ	Euler's first angle, see figure 2.
ψ	Euler's third angle, see figure 4.
$\vec{\omega} = \begin{Bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \end{Bmatrix}$	Angular velocity in principal axis coordinate system.

3. Subscripts

x, y, z	Refer to inertial cocrdinates.
1, 2, 3	Refer to body-fixed coordinates, figure 1.
n, b, t	Refer to flow-based coordinates, figure 5.
N, B, T	Refer to curvature-based coordinates, figure 6.
0	Refers to unstretched lengths.

LIST OF SYMBOLS (Cont'd)

4. Superscripts

$'$	Derivative ω/r to ℓ_0 : $\frac{d}{d\ell_0}$.
$\dot{}$	Derivative ω/r to t : $\frac{d}{dt}$.
$\rightarrow$	Vector.
$\hat{}$	Unit vector.

FOREWORD

The following report will develop a complete set of field equations for the dynamic motions of a towed cable system. These equations, when solved, constitute a dynamic simulation of a towed cable system.

During gentle maneuvers, the important forces in the governing force balances on a cable system are tension and hydrodynamic drag. In more severe maneuvers involving highly dynamic situations, however, the effects of shear, bending, torsion, extension, rotary inertia, and inertia can be important, and sometimes even mathematically necessary.

Many dynamic models for simulating the motions of a towed cable system already exist. Many of these models, however, by ignoring the above-mentioned effects, have restrictive assumptions built into them. The model represented by the equations developed here does not have these restrictions. Thus, the equations can be used either for more accurate simulations of highly dynamic situations or simply for studying the range of validity of simpler models.

A COMPLETE SET OF FIELD EQUATIONS FOR THE DYNAMIC SIMULATION OF A TOWED CABLE SYSTEM

THE COORDINATE SYSTEMS

The total field equations describing the dynamic behavior of a towed system involve four coordinate systems: 1) a fixed inertial coordinate system in which to write the translational dynamic equations, 2) a body-fixed/principal-axis coordinate system in which to write the rotational dynamic equations, 3) a flow-based coordinate system in which to write the hydrodynamic loading, and 4) a coordinate system based on the local cable curvature in which to express the bending and twisting moments.

THE BODY-FIXED COORDINATE SYSTEM $(\hat{1}, \hat{2}, \hat{3})$

Consider two orthonormal coordinate systems: a fixed inertial one $(\hat{x}, \hat{y}, \hat{z})$, and a body-fixed one $(\hat{1}, \hat{2}, \hat{3})$. (See figure 1.) Three successive rotations, through the Euler angles φ, θ, and ψ, can be performed to transform $(\hat{x}, \hat{y}, \hat{z})$ to $(\hat{1}, \hat{2}, \hat{3})$.

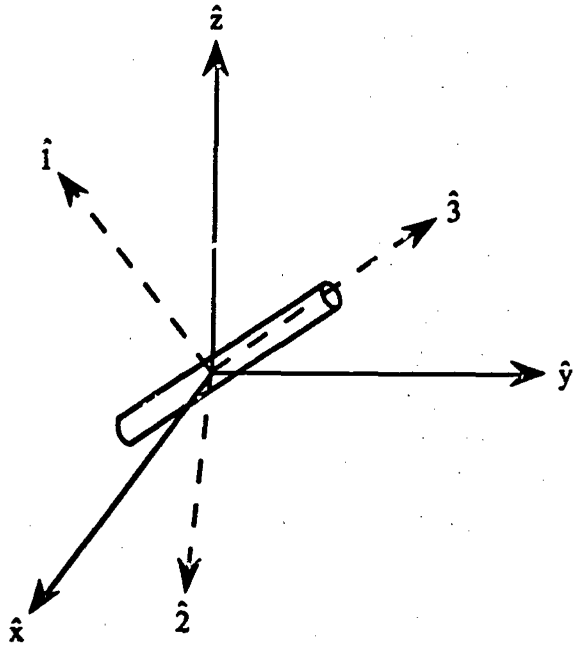
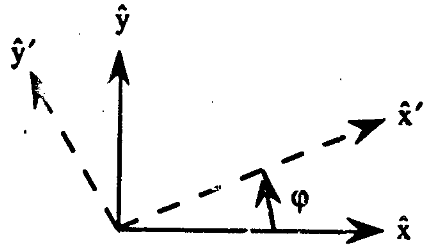


Figure 1. The Inertial and Body-Fixed Systems

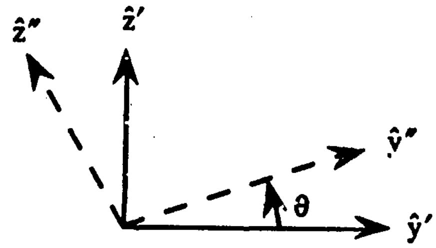
The first rotation, through the angle φ about the $\hat{z}$ axis, transforms the $(\hat{x}, \hat{y}, \hat{z})$ system to an $(\hat{x}', \hat{y}', \hat{z}')$ system where, of course, $\hat{z}$ coincides with $\hat{z}'$. The second rotation, through the angle θ about the $\hat{x}'$ axis, transforms the $(\hat{x}', \hat{y}', \hat{z}')$ system to an $(\hat{x}'', \hat{y}'', \hat{z}'')$ system where, of course, $\hat{x}''$ coincides with $\hat{x}'$. Finally, the third rotation, through the angle ψ about the $\hat{z}'$ axis, transforms the $(\hat{x}'', \hat{y}'', \hat{z}'')$ system to our body-fixed system $(\hat{1}, \hat{2}, \hat{3})$. Figures 2 through 4 show these rotations, as well as the resulting rotational matrices $\mathcal{M}_\varphi$, $\mathcal{M}_\theta$, and $\mathcal{M}_\psi$.



$$\mathcal{M}_\varphi = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 \\ -\sin\varphi & \cos\varphi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{V}_{(x'y'z')} = \mathcal{M}_\varphi \vec{V}_{(xyz)}$$

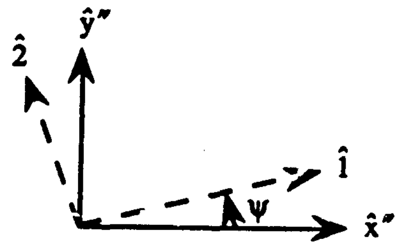
Figure 2. The φ Rotation



$$\mathcal{M}_\theta = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta & \sin\theta \\ 0 & -\sin\theta & \cos\theta \end{bmatrix}$$

$$\vec{V}_{(x''y''z'')} = \mathcal{M}_\theta \vec{V}_{(x'y'z')}$$

Figure 3. The θ Rotation



$$\mathcal{M}_\psi = \begin{bmatrix} \cos\psi & \sin\psi & 0 \\ -\sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\vec{V}_{(123)} = \mathcal{M}_\psi \vec{V}_{(x''y''z'')}$$

Figure 4. The ψ Rotation

Thus, the inertial and body-fixed coordinate systems are related as follows,

$$\bar{V}_{(1\,2\,3)} = \mathcal{M}\ \ \bar{V}_{(x\,y\,z)} \tag{1}$$

$$\mathcal{M} = \mathcal{M}_{\psi}\,\mathcal{M}_{\theta}\,\mathcal{M}_{\varphi} \tag{2}$$

$$\mathcal{M} = \begin{bmatrix} \cos\varphi\cos\psi - \sin\varphi\cos\theta\sin\psi & \sin\varphi\cos\psi + \cos\varphi\cos\theta\sin\psi & \sin\theta\sin\psi \\ -\cos\varphi\sin\psi - \sin\varphi\cos\theta\cos\psi & -\sin\varphi\sin\psi + \cos\varphi\cos\theta\cos\psi & \sin\theta\cos\psi \\ \sin\varphi\sin\theta & -\cos\varphi\sin\theta & \cos\theta \end{bmatrix} \tag{3}$$

$$\mathcal{M} = \begin{bmatrix} \hat{x}\cdot\hat{1} & \hat{y}\cdot\hat{1} & \hat{z}\cdot\hat{1} \\ \hat{x}\cdot\hat{2} & \hat{y}\cdot\hat{2} & \hat{z}\cdot\hat{2} \\ \hat{x}\cdot\hat{3} & \hat{y}\cdot\hat{3} & \hat{z}\cdot\hat{3} \end{bmatrix} \tag{4}$$

and, in equations (3) and (4), we have spelled out the components of the $\hat{1}$, $\hat{2}$, and $\hat{3}$ unit vectors in the $(\hat{x}, \hat{y}, \hat{z})$ system. However, because we will also be needing the components of $\hat{1}'$, $\hat{2}'$, and $\hat{3}'$ in the $(\hat{x}, \hat{y}, \hat{z})$ system, now is a good time to write them out, too.

COMPUTATION OF $\hat{1}'$, $\hat{2}'$, AND $\hat{3}'$

The derivatives of the cable-fixed unit vectors with respect to unstretched cable arc length will be derived here (they will be needed in the next section). By the chain rule we can write

$$\begin{aligned} \frac{\partial\hat{1}}{\partial\ell_0} &= \frac{\partial\hat{1}}{\partial\varphi}\frac{\partial\varphi}{\partial\ell_0} + \frac{\partial\hat{1}}{\partial\theta}\frac{\partial\theta}{\partial\ell_0} + \frac{\partial\hat{1}}{\partial\psi}\frac{\partial\psi}{\partial\ell_0} \\ \frac{\partial\hat{2}}{\partial\ell_0} &= \frac{\partial\hat{2}}{\partial\varphi}\frac{\partial\varphi}{\partial\ell_0} + \frac{\partial\hat{2}}{\partial\theta}\frac{\partial\theta}{\partial\ell_0} + \frac{\partial\hat{2}}{\partial\psi}\frac{\partial\psi}{\partial\ell_0} \\ \frac{\partial\hat{3}}{\partial\ell_0} &= \frac{\partial\hat{3}}{\partial\varphi}\frac{\partial\varphi}{\partial\ell_0} + \frac{\partial\hat{3}}{\partial\theta}\frac{\partial\theta}{\partial\ell_0} + \frac{\partial\hat{3}}{\partial\psi}\frac{\partial\psi}{\partial\ell_0}. \end{aligned} \tag{5}$$

Thus, finding the derivatives of the $(\hat{1}, \hat{2}, \hat{3})$ unit vectors with respect to ℓ_0 comes down to finding their derivatives with respect to the three Euler angles, φ, θ, and ψ.

From figure 4 we can write from inspection

$$\begin{aligned} \frac{\partial \hat{1}}{\partial \psi} &= \hat{2} \\ \frac{\partial \hat{2}}{\partial \psi} &= -\hat{1} \\ \frac{\partial \hat{3}}{\partial \psi} &= 0. \end{aligned} \tag{6}$$

From figure 3 we can write from inspection

$$\begin{aligned} \frac{\partial \hat{x}''}{\partial \theta} &= 0 \\ \frac{\partial \hat{y}''}{\partial \theta} &= \hat{z}'' \\ \frac{\partial \hat{z}''}{\partial \theta} &= -\hat{y}''. \end{aligned} \tag{7}$$

Using the $\mathcal{M}_\psi$ rotation matrix from figure 4, we can rewrite each double-primed coordinate system vector in equation (7) in terms of the body-fixed system unit vectors. This gives

$$\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \partial\hat{1}/\partial\theta \\ \partial\hat{2}/\partial\theta \\ \partial\hat{3}/\partial\theta \end{bmatrix} = \begin{bmatrix} 0 \\ \hat{3} \\ -\sin\psi\,\hat{1} - \cos\psi\,\hat{2} \end{bmatrix} \tag{8}$$

which, when solved, yields

$$\begin{aligned}
\frac{\partial \hat{1}}{\partial \theta} &= \sin\psi\,\hat{3} \\
\frac{\partial \hat{2}}{\partial \theta} &= \cos\psi\,\hat{3} \\
\frac{\partial \hat{3}}{\partial \theta} &= -\sin\psi\,\hat{1} - \cos\psi\,\hat{2}\,.
\end{aligned} \tag{9}$$

From figure 2, we can write from inspection

$$\begin{aligned}
\frac{\partial \hat{x}'}{\partial \varphi} &= \hat{y}' \\
\frac{\partial \hat{y}'}{\partial \varphi} &= -\hat{x}' \\
\frac{\partial \hat{z}'}{\partial \varphi} &= 0\,.
\end{aligned} \tag{10}$$

Using the $\mathcal{M}_\psi$ and $\mathcal{M}_\theta$ matrices from figures 3 and 4, we can rewrite each single-primed unit vector in equation (10) in terms of the body-fixed system unit vectors. This gives

$$\begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \cos\theta\sin\psi & \cos\theta\cos\psi & -\sin\theta \\ \sin\theta\sin\psi & \sin\theta\cos\psi & \cos\theta \end{bmatrix} \begin{bmatrix} \partial\hat{1}/\partial\varphi \\ \partial\hat{2}/\partial\varphi \\ \partial\hat{3}/\partial\varphi \end{bmatrix} = \begin{bmatrix} \cos\theta\sin\psi\,\hat{1} + \cos\theta\cos\psi\,\hat{2} - \sin\theta\,\hat{3} \\ -\cos\psi\,\hat{1} + \sin\psi\,\hat{2} \\ 0 \end{bmatrix} \tag{11}$$

which, when solved, yields

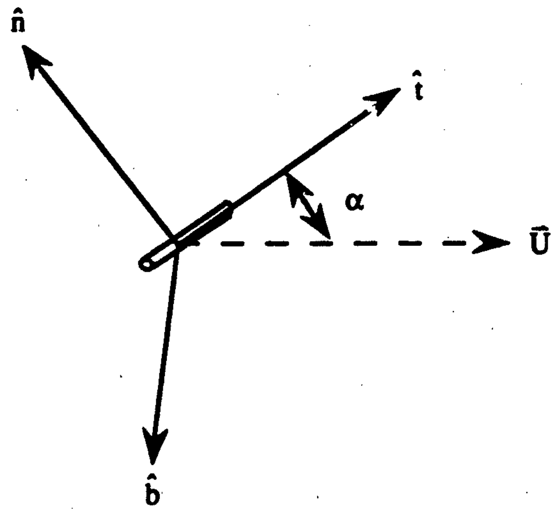
$$\begin{aligned}
\frac{\partial \hat{1}}{\partial \varphi} &= \cos\theta\,\hat{2} - \sin\theta\cos\psi\,\hat{3} \\
\frac{\partial \hat{2}}{\partial \varphi} &= -\cos\theta\,\hat{1} + \sin\theta\sin\psi\,\hat{3} \\
\frac{\partial \hat{3}}{\partial \varphi} &= \sin\theta\cos\psi\,\hat{1} - \sin\theta\sin\psi\,\hat{2}\,.
\end{aligned} \tag{12}$$

Finally, substituting equations (6), (9), and (12) into equation (5) gives us the derivatives we were after:

$$
\begin{aligned}
\frac{\partial \hat{1}}{\partial \ell_0} &= \begin{Bmatrix} 0 \\ \varphi' \cos\theta + \psi' \\ -\varphi' \sin\theta \cos\psi + \theta' \sin\psi \end{Bmatrix}_{123} \\
\frac{\partial \hat{2}}{\partial \ell_0} &= \begin{Bmatrix} -\varphi' \cos\theta - \psi' \\ 0 \\ \varphi' \sin\theta \sin\psi + \theta' \cos\psi \end{Bmatrix}_{123} \\
\frac{\partial \hat{3}}{\partial \ell_0} &= \begin{Bmatrix} \varphi' \sin\theta \cos\psi - \theta' \sin\psi \\ -\varphi' \sin\theta \sin\psi - \theta' \cos\psi \\ 0 \end{Bmatrix}_{123} .
\end{aligned}
\tag{13}
$$

THE FLOW-BASED COORDINATE SYSTEM $(\hat{n}, \hat{b}, \hat{t})$

The hydrodynamic forces on the cable will be written most easily in a coordinate system using the local flow, $\vec{U}$, and the cable to define the system. The local flow $\vec{U}$ is the cable velocity through the water and is thus the cable velocity $\vec{u}$ less any currents: $\vec{U} = \vec{u} - \vec{U}_c$. Figure 5 defines such a system.



$$\hat{n} = \hat{b} \times \hat{t}$$
$$\hat{b} = \vec{U} \times \hat{t}/U \sin\alpha$$
$$\hat{t} = \hat{3}$$
$$\vec{U} = (U, V, W)$$

Figure 5. The Flow-Based Coordinate System

In this coordinate system, the tangent vector $\hat{t}$ is defined as $\hat{3}$, the same tangent vector used in the body-fixed coordinate system. Next, using $\hat{t}$ and $\vec{U}$ to form a cable/flow plane, the binormal $\hat{b}$ is defined normal to the $\hat{t}/\vec{U}$ plane. Finally, using $\hat{t}$ and $\hat{b}$, the normal vector $\hat{n}$ is formed, completing the orthonormal set $\left(\hat{n}, \hat{b}, \hat{t}\right)$. Thus, performing the algebra indicated in figure 5, and using the results of equations (3) and (4), we can write:

$$\hat{n} = \frac{1}{|U| \sin\alpha} \begin{Bmatrix} W \sin\varphi \sin\theta \cos\theta - U \cos\theta \cos\theta - U \cos\varphi \cos\varphi \sin\theta \sin\theta \\ - V \cos\varphi \sin\varphi \sin\theta \sin\theta \\ -U \cos\varphi \sin\varphi \sin\theta \sin\theta - V \sin\varphi \sin\varphi \sin\theta \sin\theta - V \cos\theta \cos\theta \\ - W \cos\varphi \cos\theta \sin\theta \\ -V \cos\varphi \cos\theta \sin\theta - W \sin\theta \sin\theta + U \sin\varphi \cos\theta \sin\theta \end{Bmatrix}_{xyz}$$

$$\hat{b} = \frac{1}{|U| \sin\alpha} \begin{Bmatrix} V \cos\theta + W \cos\varphi \sin\theta \\ W \sin\varphi \sin\theta - U \cos\theta \\ -U \cos\varphi \sin\theta - V \sin\varphi \sin\theta \end{Bmatrix}_{xyz} \tag{14}$$

$$\hat{t} = \begin{Bmatrix} \sin\varphi \sin\theta \\ -\cos\varphi \sin\theta \\ \cos\theta \end{Bmatrix}_{xyz}$$

$$\cos\alpha = \frac{\hat{t} \cdot \vec{U}}{|U|} = (U \sin\phi \sin\theta - V \cos\phi \sin\theta + W \cos\theta)/|U|.$$

THE CURVATURE-BASED COORDINATE SYSTEM $(\hat{N}, \hat{B}, \hat{T})$

A curved line in space can be used to define three directions: tangent, normal, and binormal. Consider figure 6.

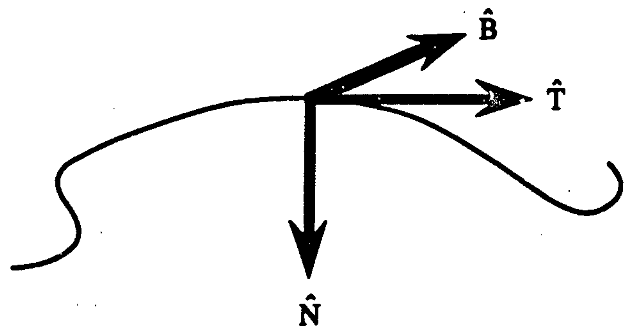


Figure 6. The Curvature-Based Coordinate System

The tangent vector $\hat{T}$ in this coordinate system is the same as that in the flow-based coordinate system; it is also defined as $\hat{3}$, the body-fixed tangent vector.

The normal vector $\hat{N}$ is defined by the curvature; in other words, by the way $\hat{T}$ changes. Locally, the curvature of the line defines a plane, the plane of curvature. $\hat{T}'$ is orthogonal to $\hat{T}$, but its magnitude is equal to the curvature, not to unity. Thus the reciprocal of the curvature, the radius of curvature ρ, is used to form a unit vector out of $\hat{T}'$. Finally, the binormal vector $\hat{B}$ is formed from the cross product of $\hat{T}$ and $\hat{N}$. This completes our orthonormal set $(\hat{N}, \hat{B}, \hat{T})$. The N/B plane is called the normal plane, the B/T plane is called the osculating plane, and the T/N plane is called the plane of curvature.

THE DYNAMIC EQUATIONS — TRANSLATION

Consider a differential section of cable of stretched length $d\ell$, as shown in figure 7.

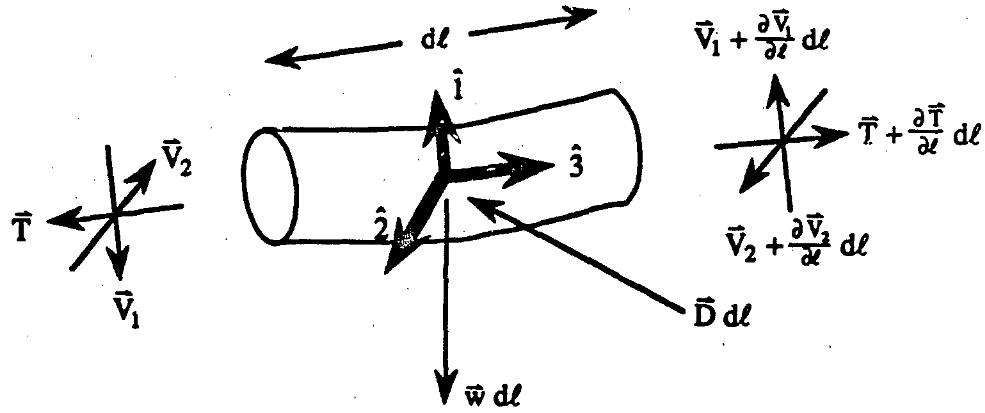


Figure 7. Force Balance on Cable Segment

The external forces acting on the cable are the weight per unit length $\vec{w}$ and the hydrodynamic load per unit length $\vec{D}$. The internal forces acting on the cable segment are, in our body-fixed coordinate system $(\hat{1}, \hat{2}, \hat{3})$, the tension $\vec{T}$, and two shears, $\vec{V}_1$ and $\vec{V}_2$. Thus, a simple force balance yields

$$\frac{\partial \vec{T}}{\partial \ell} d\ell + \frac{\partial \vec{V}_1}{\partial \ell} d\ell + \frac{\partial \vec{V}_2}{\partial \ell} d\ell - w\,\hat{z}\, d\ell + \vec{D}\, d\ell = \mu\, d\ell\, \dot{\vec{u}}. \tag{15}$$

In equation (15) we use our sign convention wherein forces are considered positive when drawn in a positive direction on a positive face. Both w, the weight per unit length, and μ, the mass per unit length, are written in terms of the stretched length, $d\ell$. We can write everything in terms of the unstretched length, $d\ell_0$, by introducing the strain, ε, and the stretch, e:

$$\varepsilon = \frac{\Delta(d\ell_0)}{d\ell_0} = \frac{d\ell - d\ell_0}{d\ell_0} = \frac{d\ell}{d\ell_0} - 1 \tag{16}$$

$$e = \frac{d\ell}{d\ell_0} = \varepsilon + 1. \tag{17}$$

Thus, we can introduce the unstretched mass and weights per unit length, μ_0 and w_0,

$$\mu = \frac{\mu_0}{e} \tag{18}$$

$$w = \frac{w_0}{e}, \tag{19}$$

and rewrite equation (15) as

$$\frac{\partial \vec{V}_1}{\partial \ell_0} + \frac{\partial \vec{V}_2}{\partial \ell_0} + \frac{\partial \vec{T}}{\partial \ell_0} - w_0\,\hat{z} + e\,\vec{D} = \mu_0\,\dot{\vec{u}} \tag{20}$$

or

$$\vec{V}_1' + \vec{V}_2' + \vec{T}' - w_0\,\hat{z} + e\,\vec{D} = \mu_0\,\dot{\vec{u}}, \tag{21}$$

where the " $'$ " refers to differentiation with respect to ℓ_0, and the " $\dot{}$ " refers to differentiation with respect to time. Equation (21) is deceptively simple because only the w_0 term and the $\dot{\vec{u}}$ term are in the inertial coordinate system $(\hat{x}, \hat{y}, \hat{z})$. Written explicitly in terms of the body-fixed and flow-based coordinate systems, equation (21) becomes

$$\begin{aligned} &V_1'\,\hat{1} + V_1\,\hat{1}' + V_2'\,\hat{2} + V_2\,\hat{2}' + T'\,\hat{3} + T\,\hat{3}' \\ &-w_0\,\hat{z} + e\,D_n\hat{n} + e\,D_t\hat{t} = \mu_0\left(\dot{u}\,\hat{x} + \dot{v}\,\hat{y} + \dot{w}\,\hat{z}\right), \end{aligned} \tag{22}$$

where D_n is the normal component of the drag force per unit length and D_t is the tangential component, both to be specified later.

To rewrite equation (22) in terms of only the fixed reference frame $(\hat{x}, \hat{y}, \hat{z})$, we must first replace the derivatives of the body-fixed unit vectors $\left(\hat{1}', \hat{2}', \hat{3}'\right)$ by their values given in equation (13). Next, we must replace all non-inertial unit vectors $\left(\hat{1}, \hat{2}, \hat{3}, \hat{n}, \hat{t}\right)$ by their values given in equations (4) and (14). The result is our three translational dynamic equations of motion:

$$F_x = m \; a_x$$

$$\begin{aligned}
& T' (\sin\varphi \sin\theta) \\
& + T (\varphi' \cos\varphi \sin\theta + \theta' \sin\varphi \cos\theta) \\
& + V_1' (\cos\varphi \cos\psi - \sin\varphi \cos\theta \sin\psi) \\
& + V_1 (-\varphi' \sin\varphi \cos\psi - \psi' \cos\varphi \sin\psi - \varphi' \cos\varphi \cos\theta \sin\psi \\
& \qquad + \theta' \sin\varphi \sin\theta \sin\psi - \psi' \sin\varphi \cos\theta \cos\psi) \\
& + V_2' (-\cos\varphi \sin\psi - \sin\varphi \cos\theta \cos\psi) \\
& + V_2 (\varphi' \sin\varphi \sin\psi - \psi' \cos\varphi \cos\psi - \varphi' \cos\varphi \cos\theta \cos\psi \\
& \qquad + \theta' \sin\varphi \sin\theta \cos\psi + \psi' \sin\varphi \cos\theta \sin\psi) \\
& + \frac{e D_n}{|U| \sin\alpha} \left(-U \cos^2\varphi \sin^2\theta - U \cos^2\theta - V \cos\varphi \sin\varphi \sin^2\theta + W \sin\varphi \cos\theta \sin\theta\right) \\
& + e D_t (\sin\varphi \sin\theta) \\
& \qquad = \mu_0 \, \dot{u} \, .
\end{aligned} \tag{23}$$

$$F_y = m \quad a_y$$

$$\begin{aligned}
&T'(-\cos\varphi \sin\theta) \\
&+ T(\varphi' \sin\varphi \sin\theta - \theta' \cos\varphi \cos\theta) \\
&+ V_1'(\sin\varphi \cos\psi + \cos\varphi \cos\theta \sin\psi) \\
&+ V_1(\varphi' \cos\varphi \cos\psi - \psi' \sin\varphi \sin\psi - \varphi' \sin\varphi \cos\theta \sin\psi \\
&\qquad - \theta' \cos\varphi \sin\theta \sin\psi + \psi' \cos\varphi \cos\theta \cos\psi) \\
&+ V_2'(-\sin\varphi \sin\psi + \cos\varphi \cos\theta \cos\psi) \\
&+ V_2(-\varphi' \cos\varphi \sin\psi - \psi' \sin\varphi \cos\psi - \varphi' \sin\varphi \cos\theta \cos\psi \\
&\qquad - \theta' \cos\varphi \sin\theta \cos\psi - \psi' \cos\varphi \cos\theta \sin\psi) \\
&+ \frac{eD_n}{|U| \sin\alpha}\left(-U \cos\varphi \sin\varphi \sin^2\theta - V \sin^2\varphi \sin^2\theta - V \cos^2\theta - W \cos\varphi \cos\theta \sin\theta\right) \\
&+ eD_t(-\cos\varphi \sin\theta) \\
&\qquad = \mu_0 \dot{v} .
\end{aligned} \tag{24}$$

$$F_z = m \quad a_z$$

$$\begin{aligned}
&T' \cos\theta \\
&+ T(-\theta' \sin\theta) \\
&+ V_1'(\sin\theta \sin\psi) \\
&+ V_1(\theta' \cos\theta \sin\psi + \psi' \sin\theta \cos\psi) \\
&+ V_2'(\sin\theta \cos\psi) \\
&+ V_2(\theta' \cos\theta \cos\psi - \psi' \sin\theta \sin\psi) \\
&+ \frac{eD_n}{|U| \sin\alpha}\left(U \sin\varphi \cos\theta \sin\theta - V \cos\varphi \cos\theta \sin\theta - W \sin^2\theta\right) \\
&+ eD_t(\cos\theta) \\
&- w_0 \\
&\qquad = \mu_0 \dot{w} .
\end{aligned} \tag{25}$$

THE DYNAMIC EQUATIONS — ROTATION

Consider a differential section of cable as shown in figure 8.

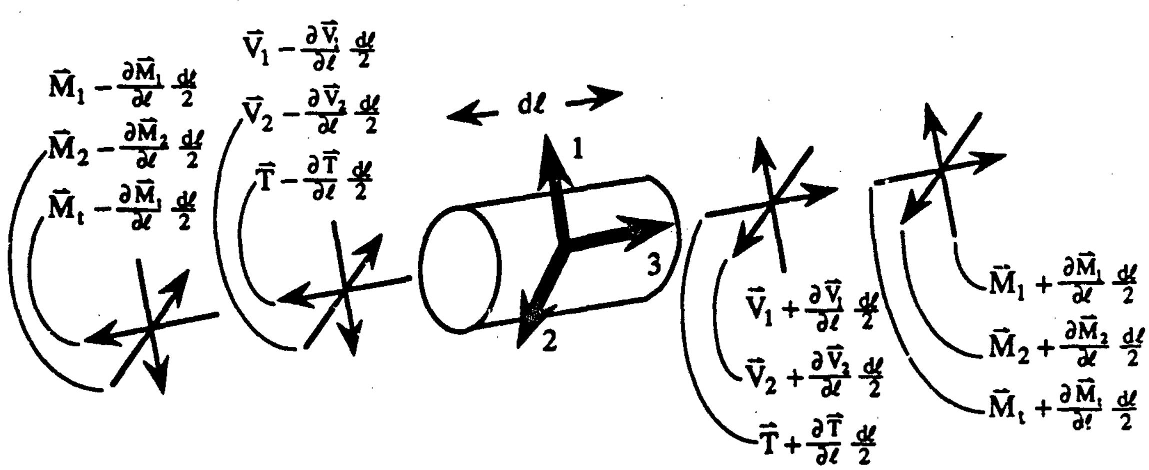


Figure 8. Moment Balance on Cable Segment

As shown in figure 8, the cable segment is subjected to the internal shearing forces and tension $(\vec{V}_1, \vec{V}_2, \text{and } \vec{T})$, as well as the internal bending and twisting moments $(\vec{M}_1, \vec{M}_2, \text{and } \vec{M}_t)$. By taking moments around the center of the segment, we need never introduce the weight and drag forces (whose torque contributions are of a higher order). In the body-fixed/principal-axis coordinate system, the dynamic equations of rotation are

$$\begin{aligned} \tau_1 &= I_1\,\dot{\omega}_1 + (I_3 - I_2)\,\omega_2\,\omega_3 \\ \tau_2 &= I_2\,\dot{\omega}_2 + (I_1 - I_3)\,\omega_3\,\omega_1 \\ \tau_3 &= I_3\,\dot{\omega}_3 + (I_2 - I_1)\,\omega_1\,\omega_2\,, \end{aligned} \tag{26}$$

where τ_i is the torque about the ith axis; I_i, the principal moment of inertia about the ith axis; and ω_i, the angular speed about the ith axis.

From figure 8, the net torque on our differential segment is

$$d\vec{\tau} = \frac{\partial \vec{M}_1}{\partial \ell} d\ell + \frac{\partial \vec{M}_2}{\partial \ell} d\ell + \frac{\partial \vec{M}_t}{\partial \ell} d\ell + V_1 d\ell\, \hat{2} - V_2 d\ell\, \hat{1} . \tag{27}$$

By introducing the unstretched length [equation (17)], equation (27) is changed to

$$d\vec{\tau} = \vec{M}_1' d\ell_0 + \vec{M}_2' d\ell_0 + \vec{M}_t' d\ell_0 + V_1 e d\ell_0 \hat{2} - V_2 e d\ell_0 \hat{1} \tag{28}$$

or

$$\vec{\tau}' = \vec{M}_1' + \vec{M}_2' + \vec{M}_t' + V_1 e \hat{2} - V_2 e \hat{1} \tag{29}$$

or

$$\vec{\tau}' = M_1' \hat{1} + M_1 \hat{1}' + M_2' \hat{2} + M_2 \hat{2}' + M_t' \hat{3} + M_t \hat{3}' + V_1 e \hat{2} - V_2 e \hat{1} . \tag{30}$$

The three principal moments of inertia of our segment of length $d\ell$ are:

$$\begin{aligned} dI_1 &= dI_2 = \tfrac{1}{4} \mu d\ell r^2 \\ dI_3 &= \tfrac{1}{2} \mu d\ell r^2 . \end{aligned} \tag{31}$$

Again, we introduce the unstretched lengths

$$\begin{aligned} I_1' &= I_2' = I_0' , \\ I_3' &= 2 I_0' , \text{ and} \\ I_0' &= \tfrac{1}{4} \mu_0 r^2 . \end{aligned} \tag{32}$$

In the body-fixed/principal-axis coordinate system, the angular velocity is written in terms of the Euler's angles as (see reference 1):

$$\vec{\omega} = \begin{Bmatrix} \dot{\varphi}\sin\theta\sin\psi + \dot{\theta}\cos\psi \\ \dot{\varphi}\sin\theta\cos\psi - \dot{\theta}\sin\psi \\ \dot{\varphi}\cos\theta + \dot{\psi} \end{Bmatrix}_{123} . \tag{33}$$

Finally, using equation (33) for the angular velocities, equation (32) for the moments of inertia, equation (30) for the torques, and equation (13) for the derivatives of the unit vectors, the governing dynamic equations of rotation (26) can be rewritten as:

$$\begin{aligned} & M_1' + M_2\,(-\varphi'\cos\theta - \psi') + M_t\,(\varphi'\sin\theta\cos\psi - \theta'\sin\psi) - e\,V_2 \\ & - I_0 \left\{\ddot{\varphi}\sin\theta\sin\psi + \ddot{\theta}\cos\psi + \dot{\varphi}^2\cos\theta\sin\theta\cos\psi \right. \\ & \qquad \left. + 2\,\dot{\varphi}\,\dot{\psi}\sin\theta\cos\psi - 2\,\dot{\theta}\,\dot{\psi}\sin\psi\right\} = 0 \end{aligned} \tag{34}$$

$$\begin{aligned} & M_1\,(\varphi'\cos\theta + \psi') + M_2' + M_t\,(-\varphi'\sin\theta\sin\psi - \theta'\cos\psi) + e\,V_1 \\ & - I_0 \left\{\ddot{\varphi}\sin\theta\cos\psi - \ddot{\theta}\sin\psi - \dot{\varphi}^2\cos\theta\sin\theta\sin\psi \right. \\ & \qquad \left. - 2\dot{\varphi}\,\dot{\psi}\sin\theta\sin\psi - 2\dot{\theta}\dot{\psi}\cos\psi\right\} = 0 \end{aligned} \tag{35}$$

$$\begin{aligned} & M_1\,(-\varphi'\sin\theta\cos\psi + \theta'\sin\psi) + M_2\,(\varphi'\sin\theta\sin\psi + \theta'\cos\psi) \\ & + M_t' - 2I_0\left\{\ddot{\varphi}\cos\theta - \dot{\varphi}\dot{\theta}\sin\theta + \ddot{\psi}\right\} = 0 . \end{aligned} \tag{36}$$

CONSTITUTIVE AND KINEMATIC RELATIONS

In the second and third sections, we developed six dynamic equations in 13 unknowns: three moments (M_1, M_2, M_t), three forces (V_1, V_2, T), three velocities (u, v, w), three angles (φ, θ, ψ), and the stretch e. In this section we will develop the seven equations needed to close the system.

CONSTITUTIVE EQUATION FOR THE TWISTING MOMENT

According to the linear theory of elasticity, the twisting moment and the twist per unit length are linearly related through the shear modulus and the polar cross-sectional moment:

$$M_t = GI_P \frac{\partial \psi}{\partial \ell_0}. \tag{37}$$

If we assume that the cable is not torque-balanced, then we can write that the twisting moment is also linearly related to the strain $\varepsilon = e - 1$:

$$M_t = A_4 \psi' + A_3 (e - 1). \tag{38}$$

CONSTITUTIVE EQUATION FOR THE TENSION

Again, according to the linear theory of elasticity, the tension and the strain are linearly related through Young's Modulus and the cross-sectional area:

$$T = EA\varepsilon.$$

Again, if we assume that the cable is not torque-balanced, then we can write that the tension is also linearly related to the twist:

$$T = A_1 (e - 1) + A_2 \psi' . \tag{39}$$

CONSTITUTIVE EQUATIONS FOR THE BENDING MOMENTS

According to pure bending theory, which we will use despite the fact that shears are present, the bending moment is linearly related to the bending rigidity (EI), and inversely related to the radius of curvature (ρ). Figure 9 shows this relationship in terms of the $(\hat{N}, \hat{B}, \hat{T})$ coordinate system, the coordinate system based on the local curvature, developed in the first section.

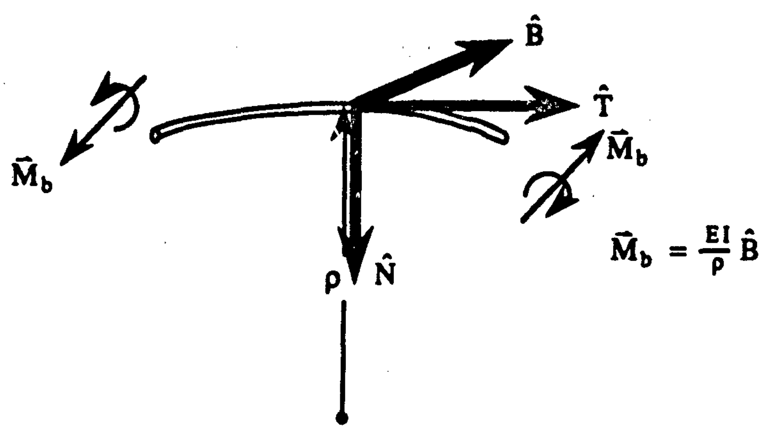


Figure 9. Pure Bending

Because the bending moment is defined as being in the binormal direction, we can write, from figures 6 and 9,

$$\vec{M}_B = \frac{EI_B}{\rho}\hat{B} = EI_B\,\hat{T}\times\frac{\partial}{\partial\ell}\hat{T} \tag{40}$$

or

$$e\vec{M}_B = EI_B\,\hat{T}\times\hat{T}'. \tag{41}$$

Using equation (13) for $\hat{T}' = \partial\hat{3}/\partial\ell_0$, we can write

$$\hat{T}\times\hat{T}' = \begin{Bmatrix} -(-\varphi'\sin\theta\sin\psi - \theta'\cos\psi) \\ \varphi'\sin\theta\cos\psi - \theta'\sin\psi \\ 0 \end{Bmatrix}_{123} \tag{42}$$

or

$$e\vec{M}_B = EI_B \begin{Bmatrix} \varphi'\sin\theta\sin\psi + \theta'\cos\psi \\ \varphi'\sin\theta\cos\psi - \theta'\sin\psi \\ 0 \end{Bmatrix}_{123}, \tag{43}$$

where the cross-sectional bending moment I_B is for bending around the $\hat{B}$ axis and is, for a homogeneous cable, $\pi R^4/4$. Thus, the constitutive equations for the two bending moments are

$$eM_1 = EI_B(\varphi'\sin\theta\sin\psi + \theta'\cos\psi) \tag{44}$$

and

$$eM_2 = EI_B(\varphi' \sin\theta \cos\psi - \theta' \sin\psi). \tag{45}$$

KINEMATIC RELATIONS

The (x, y, z) coordinates of a point on the cable, and the (φ, θ, ψ) angles at that point, are independent. However, the way in which the coordinates change along the cable is determined by the orientation of the cable, by the angles. Specifically, the cable coordinates change in the direction of the tangent vector

$$\frac{\partial \vec{r}}{\partial \ell} = \hat{3}, \tag{46}$$

which, written out by components, gives us three kinematic equations:

$$x' = \quad e \sin\varphi \sin\theta, \tag{47}$$

$$y' = -e \cos\varphi \sin\theta, \tag{48}$$

and

$$z' = \quad e \cos\theta. \tag{49}$$

SUMMARY

In this report we developed a complete set of field equations describing the dynamic behavior of a towed cable and/or array. The cable has bending stiffness, torsional stiffness, extensional stiffness, inertia, and viscous drag. The viscous drag (as well as added mass) is left as a generic term to be defined by the users according to their specific needs.

The complete set of field equations consists of 13 nonlinear partial differential equations and is repeated here for conciseness:

1. $$F_x = m \; a_x$$

$$\begin{aligned}
&T'(\sin\varphi\sin\theta) \\
&+T(\varphi'\cos\varphi\sin\theta+\theta'\sin\varphi\cos\theta) \\
&+V_1'(\cos\varphi\cos\psi-\sin\varphi\cos\theta\sin\psi) \\
&+V_1(-\varphi'\sin\varphi\cos\psi-\psi'\cos\varphi\sin\psi-\varphi'\cos\varphi\cos\theta\sin\psi \\
&\qquad+\theta'\sin\varphi\sin\theta\sin\psi-\psi'\sin\varphi\cos\theta\cos\psi) \\
&+V_2'(-\cos\varphi\sin\psi-\sin\varphi\cos\theta\cos\psi) \\
&+V_2(\varphi'\sin\varphi\sin\psi-\psi'\cos\varphi\cos\psi-\varphi'\cos\varphi\cos\theta\cos\psi \\
&\qquad+\theta'\sin\varphi\sin\theta\cos\psi+\psi'\sin\varphi\cos\theta\sin\psi) \\
&+\frac{eD_n}{|U|\sin\alpha}\left(-U\cos^2\varphi\sin^2\theta-U\cos^2\theta-V\cos\varphi\sin\varphi\sin^2\theta+W\sin\varphi\cos\theta\sin\theta\right) \\
&+eD_t(\sin\varphi\sin\theta) \\
&\qquad\qquad=(\mu_0+\mu_a)\,\dot{u}\,.
\end{aligned} \tag{50}$$

2. $$F_y = m \; a_y$$

$$\begin{aligned}
&T'(-\cos\varphi \sin\theta) \\
&+ T(\varphi' \sin\varphi \sin\theta - \theta' \cos\varphi \cos\theta) \\
&+ V_1'(\sin\varphi \cos\psi + \cos\varphi \cos\theta \sin\psi) \\
&+ V_1(\varphi' \cos\varphi \cos\psi - \psi' \sin\varphi \sin\psi - \varphi' \sin\varphi \cos\theta \sin\psi \\
&\qquad - \theta' \cos\varphi \sin\theta \sin\psi + \psi' \cos\varphi \cos\theta \cos\psi) \\
&+ V_2'(-\sin\varphi \sin\psi + \cos\varphi \cos\theta \cos\psi) \\
&+ V_2(-\varphi' \cos\varphi \sin\psi - \psi' \sin\varphi \cos\psi - \varphi' \sin\varphi \cos\theta \cos\psi \\
&\qquad - \theta' \cos\varphi \sin\theta \cos\psi - \psi' \cos\varphi \cos\theta \sin\psi) \\
&+ \frac{eD_n}{|U| \sin\alpha}\left(-U \cos\varphi \sin\varphi \sin^2\theta - V \sin^2\varphi \sin^2\theta - V \cos^2\theta - W \cos\varphi \cos\theta \sin\theta\right) \\
&+ eD_t(-\cos\varphi \sin\theta) \\
&\qquad\qquad = (\mu_0 + \mu_a)\,\dot{v}\,.
\end{aligned} \tag{51}$$

3. $$F_z = m \; a_z$$

$$\begin{aligned}
&T' \cos\theta \\
&+ T(-\theta' \sin\theta) \\
&+ V_1'(\sin\theta \sin\psi) \\
&+ V_1(\theta' \cos\theta \sin\psi + \psi' \sin\theta \cos\psi) \\
&+ V_2'(\sin\theta \cos\psi) \\
&+ V_2(\theta' \cos\theta \cos\psi - \psi' \sin\theta \sin\psi) \\
&+ \frac{eD_n}{|U| \sin\alpha}\left(U \sin\varphi \cos\theta \sin\theta - V \cos\varphi \cos\theta \sin\theta - W \sin^2\theta\right) \\
&+ eD_t(\cos\theta) \\
&- w_0 \\
&\qquad\qquad = (\mu_0 + \mu_a)\,\dot{w}\,.
\end{aligned} \tag{52}$$

4. $$\begin{aligned}
&M_1' + M_2(-\varphi' \cos\theta - \psi') + M_t(\varphi' \sin\theta \cos\psi - \theta' \sin\psi) - eV_2 \\
&- I_0\left\{\ddot{\varphi} \sin\theta \sin\psi + \ddot{\theta} \cos\psi + \dot{\varphi}^2 \cos\theta \sin\theta \cos\psi \right. \\
&\qquad \left. + 2\,\dot{\varphi}\,\dot{\psi} \sin\theta \cos\psi - 2\,\dot{\theta}\,\dot{\psi} \sin\psi\right\} = 0\,.
\end{aligned} \tag{53}$$

5. $$M_1(\varphi'\cos\theta+\psi')+M_2'+M_t(-\varphi'\sin\theta\sin\psi-\theta'\cos\psi)+eV_1$$
$$-I_0\left\{\ddot{\varphi}\sin\theta\cos\psi-\ddot{\theta}\sin\psi-\dot{\varphi}^2\cos\theta\sin\theta\sin\psi\right.$$
$$\left.-2\dot{\varphi}\dot{\psi}\sin\theta\sin\psi-2\dot{\theta}\dot{\psi}\cos\psi\right\}=0. \tag{54}$$

6. $$M_1(-\varphi'\sin\theta\cos\psi+\theta'\sin\psi)+M_2(\varphi'\sin\theta\sin\psi+\theta'\cos\psi)$$
$$+M_t'-2I_0\left\{\ddot{\varphi}\cos\theta-\dot{\varphi}\dot{\theta}\sin\theta+\ddot{\psi}\right\}=0. \tag{55}$$

7. $$M_t = A_4\,\psi'+A_3(e-1). \tag{56}$$

8. $$T = A_1(e-1)+A_2\,\psi'. \tag{57}$$

9. $$eM_1 = EI_B(\varphi'\sin\theta\sin\psi+\theta'\cos\psi). \tag{58}$$

10. $$eM_2 = EI_B(\varphi'\sin\theta\cos\psi-\theta'\sin\psi). \tag{59}$$

11. $$u' = \frac{d}{dt}(e\sin\varphi\sin\theta). \tag{60}$$

12. $$v' = -\frac{d}{dt}(e\cos\varphi\sin\theta). \tag{61}$$

13. $$w' = \frac{d}{dt}(e\cos\theta). \tag{62}$$

The 13 unknowns are the 3 forces (V_1, V_2, T), 3 moments (M_1, M_2, M_t), 3 velocities (u, v, w), 3 angles (φ, θ, ψ), and the stretch e. Written as such, the 13 equations (50-62) are first order in space and second order in time. By introducing the 3 angular velocities,

$$\chi = \dot{\varphi}, \tag{63}$$

$$\iota = \dot{\theta}\text{, and} \tag{64}$$

$$\omega = \dot{\psi}, \tag{65}$$

we would increase our system to 16 equations in 16 unknowns and become first order in space and time.

Finally, because these equations will potentially be used to study the higher order effects of rigidity and torsion, we can simplify these equations for comparison and reduce them to the more standard six unknown systems written here in equations (66) through (71):

$$\begin{aligned}
&T'(\sin\varphi\sin\theta) \\
&+T(\varphi'\cos\varphi\sin\theta + \theta'\sin\varphi\cos\theta) \\
&+\frac{D_n}{|U|\sin\alpha}\left(-U\cos^2\varphi\sin^2\theta - U\cos^2\theta - V\cos\varphi\sin\varphi\sin^2\theta + W\sin\varphi\cos\theta\sin\theta\right) \\
&\qquad + D_t\sin\varphi\sin\theta = (\mu_0 + \mu_a)\,\dot{u}
\end{aligned} \tag{66}$$

$$\begin{aligned}
&T'(-\cos\varphi\sin\theta) \\
&+T(\varphi'\sin\varphi\sin\theta - \theta'\cos\varphi\cos\theta) \\
&+\frac{D_n}{|U|\sin\alpha}\left(-U\cos\varphi\sin\varphi\sin^2\theta - V\sin^2\varphi\sin^2\theta - V\cos^2\theta - W\cos\varphi\cos\theta\sin\theta\right) \\
&\qquad + D_t(-\cos\varphi\sin\theta) = (\mu_0 + \mu_a)\,\dot{v}
\end{aligned} \tag{67}$$

$$\begin{aligned}
&T'\cos\theta + T(-\theta'\sin\theta) \\
&+\frac{D_n}{|U|\sin\alpha}\left(U\sin\varphi\cos\theta\sin\theta - V\cos\varphi\cos\theta\sin\theta - W\sin^2\theta\right) \\
&\qquad + D_t\cos\theta - w_0 = (\mu_0 + \mu_a)\,\dot{w}
\end{aligned} \tag{68}$$

$$\dot{u} = \frac{d}{dt}(\sin\varphi\sin\theta) \tag{69}$$

$$\dot{v} = -\frac{d}{dt}(\cos\varphi \sin\theta) \tag{70}$$

$$\dot{w} = \frac{d}{dt}(\cos\theta). \tag{71}$$

The solutions to these equations will be the subject of a subsequent report.

REFERENCE

1. S. H. Crandall et al., *Dynamics of Mechanical and Electromechanical Systems*, Krieger Publishing Company, Malabar, FL, 1968, p. 226.

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